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Instantaneous power dissipation in class-B power transistor

Using complex amplifier load impedance

$$\boldsymbol{Z}=Z_r+jZ_i=|\boldsymbol{Z}|e^{j\phi_z} \quad \text{where} \quad j=\sqrt{-1} \qquad 1.$$

And complex amplifier output voltage/current

$$I(t)=\Re(\boldsymbol{I}e^{j\omega t})=\Re(|\boldsymbol{I}|e^{\phi_I+j\omega t})=|\boldsymbol{I}|\cos(\phi_I+\omega t) \qquad 2.$$

$$V(t)=\Re(\boldsymbol{V}e^{j\omega t})=\Re(|\boldsymbol{V}|e^{\phi_V+j\omega t})=|\boldsymbol{V}|\cos(\phi_V+\omega t) \qquad 3.$$

Defining the complex output current to be

$$\boldsymbol{I}=-jI_p \qquad 4.$$

This means that

$$I(t)=I_p\sin(\omega t) \qquad 5.$$

Ohms law in complex units is simply

$$\boldsymbol{V}=\boldsymbol{I}\boldsymbol{Z} \qquad 6.$$

Hence the complex output voltage is

$$\boldsymbol{V}=-jI_p\boldsymbol{Z} \qquad 7.$$

Which means that

$$V(t)=I_p|\boldsymbol{Z}|\cos(\phi_Z+\omega t-\pi/2)=I_p|\boldsymbol{Z}|\sin(\phi_Z+\omega t) \qquad 8.$$

Or

$$V(t)=V_p\sin(\phi_Z+\omega t) \qquad 9.$$

where $V_p=I_p|\boldsymbol{Z}|$

The instantaneous power dissipated in the output transistor is

$$P(\omega t)=[V_r-V(t)]I(t) \qquad 10.$$

Or

$$P(\omega t)=[V_r-V_p\sin(\phi_Z+\omega t)]I_p\sin(\omega t) \qquad 11.$$

Writing this in terms of voltages only

$$P(\omega t)=[V_r-V_p\sin(\phi_Z+\omega t)]\frac{V_p}{|\boldsymbol{Z}|}\sin(\omega t) \qquad 12.$$

Expanding

$$P(\omega t)=\frac{V_rV_p}{|\boldsymbol{Z}|}\sin(\omega t)-\frac{V_p^2}{|\boldsymbol{Z}|}\sin(\omega t)\sin(\phi_Z+\omega t) \qquad 13.$$

Differentiating

$$\frac{dP(\omega t)}{d\omega t}=\frac{V_r V_p}{|\boldsymbol{Z}|}\cos(\omega t)-\frac{V_p^2}{|\boldsymbol{Z}|}\left[\cos(\omega t)\sin(\phi_Z+\omega t)+\sin(\omega t)\cos(\phi_Z+\omega t)\right] \qquad 14.$$

Maximum and minimum dissipation occurs when $V_r=V_p$ and $\frac{dP(\omega t)}{d\omega t}=0$. I.e.

$$\cos(\omega t)=\cos(\omega t)\sin(\phi_Z+\omega t)+\sin(\omega t)\cos(\phi_Z+\omega t) \qquad 15.$$

$$\cos(\omega t)=\sin(2\omega t+\phi_z) \qquad 16.$$

Due to the symmetry of the cosine function this has several solutions

$$\cos(\omega t)=\sin(2\omega t+\phi_z)\,,\ \cos(-\omega t)=\sin(2\omega t+\phi_z)\ \text{and}\ \cos(2\pi-\omega t)=\sin(2\omega t+\phi_z) \qquad 17.$$

These can be written as

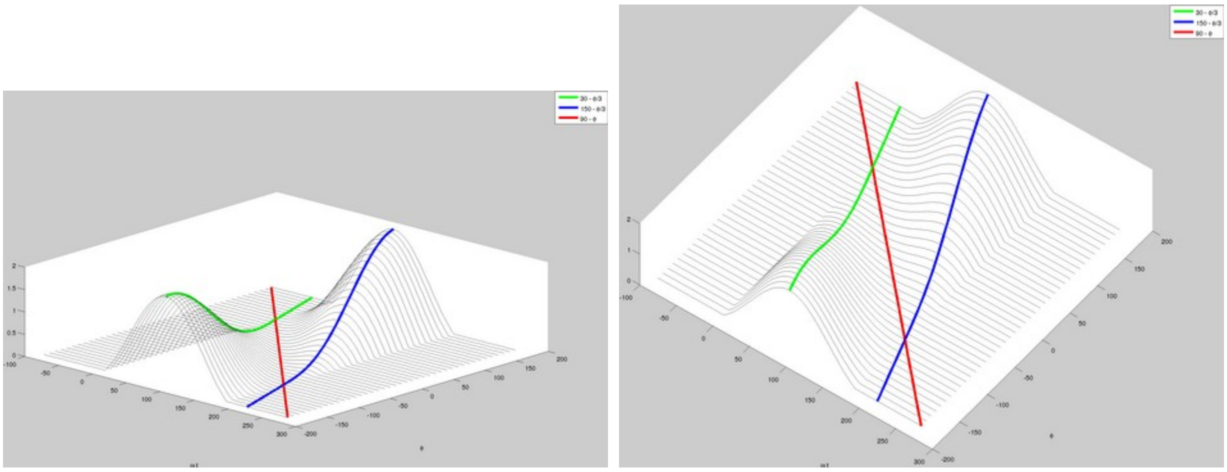
$$\sin(\omega t+\pi/2)=\sin(2\omega t+\phi_z)\,,\ \sin(-\omega t+\pi/2)=\sin(2\omega t+\phi_z)\ \text{and}\ \sin(-\omega t+\pi 5/2)=\sin(2\omega t+\phi_z) \quad 18.$$

$$\omega t=\frac{\pi}{2}-\phi_z\,,\ \omega t=\frac{\pi}{6}-\frac{\phi_z}{3}\ \text{and}\ \omega t=\frac{5}{6}\pi-\frac{\phi_z}{3} \qquad 19.$$

Or in degrees

$$\omega t=90^o-\phi_z\,,\ \omega t=30^o-\frac{\phi_z}{3}\ \text{and}\ \omega t=150^o-\frac{\phi_z}{3} \qquad 20.$$

To check this we plot equation 13 for the case when $V_p=V_r=|\boldsymbol{Z}|=1$



From this you can see that the expression $\omega t=90^o-\phi_z$ gives the minimum power dissipation phase, $\omega t=30^o-\frac{\phi_z}{3}$ gives the maximum dissipation phase when $\phi_z<0$, and $\omega t=150^o-\frac{\phi_z}{3}$ gives the maximum when $\phi_z>0$.

<u>Average power dissipation in class B amplifier output transistor</u>

Integrate over instantaneous power for the on half cycle of the transistor:

$$P_{ave}=\int_{\omega t=0}^{\pi}\frac{1}{\pi}P(\omega t)\,d\,\omega t=\frac{2V_r V_p}{\pi|\boldsymbol{Z}|}-\frac{V_p^2\cos(\phi_z)}{2|\boldsymbol{Z}|} \qquad 21.$$

<u>EPDR</u>

EPDR gives the equivalent resistance that would dissipate the same peak power for the situation when $V_p = V_r$.

Inserting this into equation 13 results in

$$P(\omega t) = \frac{V_p^2}{|\boldsymbol{Z}|}[1 - \sin(\phi_Z + \omega t)]\sin(\omega t) \qquad 22.$$

The instantaneous dissipation function has some nice symmetry so we can use just the maxima line for positive phase $\phi_z > 0$ and write the peak dissipation as

$$P_{pk} = \frac{V_p^2}{|\boldsymbol{Z}|}\left[1 - \sin\left(\frac{5}{6}\pi + \frac{2}{3}|\phi_z|\right)\right]\sin\left(\frac{5}{6}\pi - \frac{|\phi_z|}{3}\right) \qquad 23.$$

For a resistor this expression simplifies to

$$P_{Rpk} = \frac{V_p^2}{R}\left[1 - \sin\left(\frac{5}{6}\pi\right)\right]\sin\left(\frac{5}{6}\pi\right) \qquad 24.$$

Equating the resistor expression and the complex impedance expression we find the expression for EPDR:

$$R = |\boldsymbol{Z}|\frac{\left[1 - \sin\left(\frac{5}{6}\pi\right)\right]\sin\left(\frac{5}{6}\pi\right)}{\left[1 - \sin\left(\frac{5}{6}\pi + \frac{2}{3}|\phi_z|\right)\right]\sin\left(\frac{5}{6}\pi - \frac{|\phi_z|}{3}\right)} \qquad 25.$$