# Georg Simon Ohm and Ohm's Law

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[40] O. Fulton, U.S. Patent 1 985 084.
[41] C. R. Jones, *Facsimile*. New York: Murray Hill, 1949.
[42] *Post Office Elec. Eng. J.*, vol. 21, 1930.
[43] *Science News Letter*, Sept. 3, 1938.
[44] C. J. Young, "Equipment & methods developed for broadcast facsimile service," *RCA Rev.*, vol. 2, pp. 379–395, Apr. 1938.
[45] *Saturday Evening Post*, "Will your newspaper come by radio?," Nov. 13, 1946.
[46] *Better Homes & Gardens*, "All they know they read on the radio," Feb. 1947.
[47] *Editor & Publisher*, Feb. 15, 1947.
[48] *IEEE Spectrum*, vol. 16, p. 63, Oct. 1979; also, *IEEE Spectrum*, vol. 17, pp. 26–35, Mar. 1980.
[49] *IEEE Spectrum*, vol. 16, pp. 42, 197, Oct. 1979.

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# Georg Simon Ohm and Ohm's Law

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***Abstract*—A short biographical sketch of Ohm and a discussion of his experimental and theoretical work on electrical conduction in metals is given. Contemporaneous electrical science is briefly reviewed as it relates to Ohm's law, and the law is examined from a modern viewpoint.**

## I. INTRODUCTION

OHM'S law is perhaps the most popularly known of all the laws, theories, and principles of electrical sciences. The law forms the starting point in the study of electrical networks, and it has reserved, forever, a place of honor for Georg Simon Ohm (1789–1854) in the history of science. The year 1976 marked the 150th anniversary of Ohm's law.

The major objective of this paper is to describe for the engineering community, exactly what Ohm achieved. It seems that there is a lack of understanding as to precisely what his contribution was, apparently because many of the published studies on Ohm's work are addressed to historians of science, with a majority of these in German, and in the terminology of Ohm's time, thus making it difficult to understand his work. This might also account for some of the common erroneous impressions concerning Ohm's work, for example, that it was purely (or at first) theoretical [1] and that Ohm's law was first announced in his book published in 1827 [2]. An examination of Ohm's law with a modern perspective, and with the benefit of hindsight, will also help understand its limitations, generalizations, and ultimate basis in the fundamental laws of physics.

### *What is Ohm's Law?*

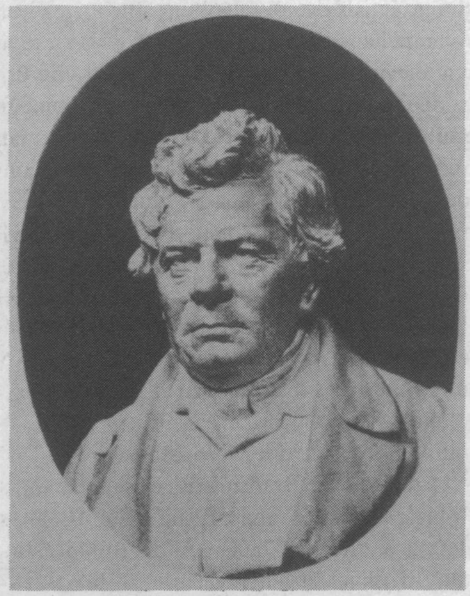

Fig. 1. Portrait of G. S. Ohm, from a bust by Rümann.

In modern terminology, Ohm's law states that the current $I$ flowing through a conductor is proportional to the voltage $V$ applied across it:

$$I = V/R. \qquad (1)$$

Ohm initially called the constant of proportionality $R$ "reduced length," but renamed it "resistance" shortly thereafter. Ohm obtained this result both experimentally and theoretically. Discovery of the manner in which $I$ depends on the area of cross section $A$ and length $l$ of the conductor, for a fixed $V$,

$$I \propto A/l \qquad (2)$$

cannot be ascribed to Ohm. Although Ohm was the first person to combine (1) and (2) to obtain

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$$I = \sigma A V/l, \tag{3}$$

it seems appropriate to give the title "Ohm's law" to (1) rather than to (3). As will be seen in Section V, a knowledge of the $A$ and $l$ dependence on the right-hand side of (2) is older than (1); its experimental demonstration can be traced back in part to Davy, and its intuitive plausibility to Cavendish who knew the effect of connecting conductors in series and parallel [3]. Ohm's contribution included identifying the numerator on the right-hand side of (1) with potential difference, and recognizing the existence of a potential gradient in the conductor.

There is a second law associated with Ohm's name in physiological acoustics, to which Ohm devoted some of his later years. It states that human ears perceive sound in terms of its individual frequency components, in effect carrying out a Fourier analysis [4]. The subsequent discussions do not refer to this, "the other Ohm's law."

## II. A BIOGRAPHICAL SKETCH

### *Early Life*

Georg Simon Ohm (christened Johann Simon at birth) was born on March 16, 1789, at Erlangen, Bavaria. His father, Johann Wolfgang Ohm, a locksmith with interests in philosophy and mathematics, encouraged and helped Georg and his younger brother, Martin, to study mathematics. Georg left the local *gymnasium* (a preuniversity school) at 16 to attend the University of Erlangen, where he studied for three terms before leaving to teach at Gottstadt in Switzerland. He returned to the University at Erlangen after living in Neuchâtel in Switzerland for some time and received his doctoral degree in 1811. After a period as a *privatdozent* (an unsalaried instructor) there, he became a teacher at a secondary school in Bamberg, where he published his first paper on teaching geometry. Four years later, he joined the *gymnasium* at Cologne as a senior master of mathematics and physics. His ambition for a university career prompted him to turn to research and he began experimenting with electrical batteries and current flow, using instruments that he made himself. The most detailed and illustrated account of Ohm's early life in English is given by Appleyard [5].

### *Formulation of Ohm's Law*

Ohm published the results of his experiments on electrical conduction for the first time in 1825, at the age of 36, simultaneously in *Journal für Chemie und Physik* (also called *Schweigger Journal*) and in *Annalen der Physik* (also called *Poggendorff's Annalen*). This paper [6] described the results of his measurement of the effect of the length of a conductor on the magnetic effect of current flowing through it. On the basis of the results, Ohm at first deduced a logarithmic empirical relationship between conductor length and current. The experimental results themselves are not totally incorrect despite the limitations of his apparatus and will be discussed later. The inaccuracy arose primarily from the polarization effects influencing the internal resistance of the battery in present terminology. Ohm himself realized the inadequacy of his formula and pointed out [7] that the relationship fails for a very long conductor (when its resistance is comparable to the internal resistance of the battery). Poggendorff, the Editor of *Annalen der Physik*, added a note to the published paper, suggesting that the experiment be repeated with a thermocouple replacing the battery as a constant-output source.

Ohm continued his experiments and published five subsequent papers and short notes between 1825 and 1827. He followed Poggendorff's suggestion and by 1826 formulated the relationship now known as Ohm's law [8]. These experiments will be described later.

Ohm then turned to a theoretical derivation of his empirical result. He took a leave of absence from the *gymnasium* and spent a year working on the monograph [9] *Die Galvanische Kette, Mathematisch Bearbeitet* (*The Galvanic Chain, Mathematically Treated*) published in 1827. (This 126 page book, the best known of Ohm's writings, was translated into English by William Francis and published in London in 1841 in *Scientific Memoirs* [10].) Confident of his achievement, Ohm sent a copy of this book, and his resignation from Cologne *Gymnasium*, to the Prussian Ministry of Education, expecting appointment to a university professorship. Instead, he received an offer of a position at the Artillery School in Berlin as a mathematics teacher. At the same time, his work failed to receive the acclaim of contemporary physicists, and was even criticized by Pohl. Discouraged from replying to this criticism by several journal editors, Ohm was bitter and disappointed. He spent the next six years teaching in Berlin and continued to work on electrical circuits.

### *Later Years and Recognition*

Recognition came to Ohm in 1833 when he was appointed a Professor at the Nuremberg Technical College. His work received more attention in England; the Royal Society of London awarded him the Copley Medal for his contributions, and appointed him a foreign member of the society in 1842, an honor received by only one German (Gauss) prior to this date. Finally, in 1849, his lifelong ambition was fulfilled when, at the age of 60, he was appointed Professor of Physics at the University of Munich. He died on July 6, 1854, at the age of 65. In his honor, the International Electrical Congress held at Paris in 1881 adopted the ohm as the unit of resistance.

### *Scientific Accomplishments*

Although Ohm's scientific career began with his studies of electrical conduction, he made contributions to several other subjects, including acoustics (1839-1843), molecular physics, and polarized light (1852-1853). He was considered an outstanding teacher and completed an elementary textbook in physics in 1854, shortly before his death. A compilation of his 23 papers and monographs was prepared by Eugen Lommel, Professor of Physics at the University of Munich, in 1892, subsequent to the centenary of Ohm's birth [11]. A second collection of his papers appeared on the occasion of the centenary of Ohm's law [12].

## III. EXPERIMENTAL WORK ON OHM'S LAW

Ohm carried out several experimental studies, but only those published before his book and leading to Ohm's law will be mentioned here.

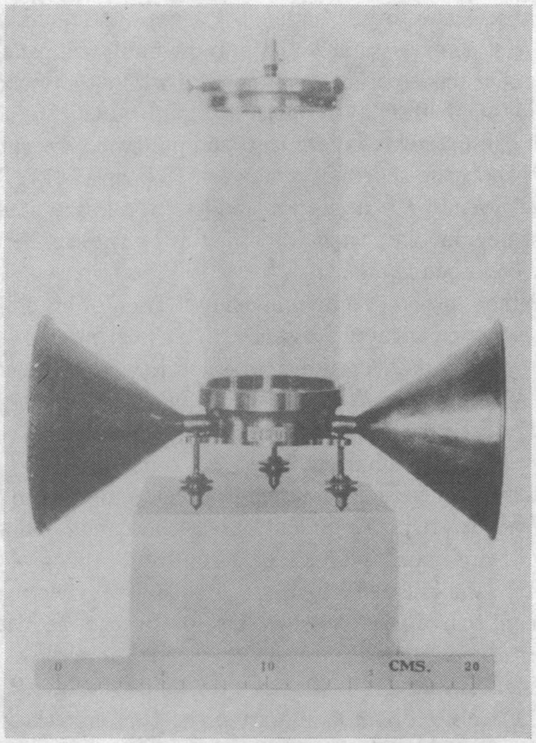

Fig. 2. Thermogalvanometer used by Ohm. It contains two rectangular bars, probably one of bismuth and the other of antimony, one above the other, at a distance sufficient to allow the lower magnet of an astatic needle to swing between them. The ends of the bars are soldered together, and the upper bar has a longitudinal saw-cut through which the lower suspended magnet can pass to its proper level. The funnel-shaped reflectors are directed towards the solderings.

### *Experiments with Voltaic Cell*

In 1825, Ohm published his first experimental study, entitled "Preliminary Notice of the Law According to which Metals Conduct Contact Electricity [6]." In this paper, he observed the effect of the length of a wire upon the "loss of force" (to be defined below), a term which has been interpreted differently by various historians as potential drop [13], current, and change in current [14].

Ohm's apparatus consisted of a wet cell as the source, across which conductors of different lengths could be connected. The effect of the length was measured by a torsion balance, rather than the more common "multiplier" (or galvanometer). Ohm himself constructed a torsion balance, consisting of a magnetic needle suspended over the conductor from a support (the torsion head) by a thin wire. When deflected, the needle was brought back to the "zero" position by turning of the torsion head and torsion of the suspension was therefore a measure of the magnetic force. The "loss of force" owing to a wire of length $x$ was simply the fractional reduction in torsion when that wire replaced a short, thick, "standard" wire.

In present-day terminology, the loss of force measured by Ohm is the same as the normalized change in current $\Delta I = (I_s - I_w)/I_s$, where $I_w$ and $I_s$ are the currents flowing when the experimental and the standard wires are used. For a battery with an open-circuit voltage $V_o$ and internal resistance $R_i$, the "loss of force" is

$$\Delta I = \frac{I_s - I_w}{I_s} = \frac{R_w - R_s}{R_i + R_w} \tag{4}$$

where $R_w$ and $R_s$ are the resistances of the experimental and standard wires. If $R_i$ is large and $R_s$ is small compared to $R_w$, as was the case in his apparatus, the loss of force reduces to $(R_w/R_i)$, and hence is linearly proportional to the wire length $x$. Note that Ohm's use of the normalized quantity $\Delta I$ as the observed variable eliminated the effect of the actual value of $V_o$ from his results.

The major experimental difficulty in this arrangement is due to the polarization of the cell. Ohm averaged the "standard torsion" (measured with the standard wire connected) before and after the experimental wire was connected. As the battery became weaker, the standard torsion also changed and Ohm calculated $\Delta I$ for a fixed standard torsion by interpolation. He found that the resulting data relating $\Delta I$ to $x$ could be represented by the empirical formula

$$\Delta I = m \log \left(1 + \frac{x}{a}\right) \tag{5}$$

where $m$ and $a$ are empirical constants. Ohm repeated the experiment several times to show that the constant $a$ depended on the "lengths of other elements in the circuit" (i.e., upon $R_i$). The other constant $m$ depended on many quantities, including $a$. When $R_i$ is large (i.e., in the limit of large $a$), (5) reduces to $\Delta I = mx/a$, which is the correct linear relationship.

In a second set of experiments, Ohm compared the conductivities of different metals by means of the same equipment. With wires of different metals having the same diameter, he adjusted wire lengths until they all resulted in the same torsion, thus eliminating the need for a standard wire.

### *Experiments with Thermocouple*

In his subsequent experiments, which led to the correct formulation of Ohm's law, Ohm followed Poggendorff's suggestion. He replaced the wet cell by a copper-bismuth thermocouple, with its two junctions maintained in boiling water and melting ice. The rest of the equipment, including the torsion balance and wires of different lengths, remained essentially the same. However, instead of the "loss of force," Ohm now selected the torsion $X$ itself as the dependent variable. This selection also eliminated the need for a "standard" wire.

He observed that his data relating the torsion $X$ and the wire length $x$ could be represented by the formula

$$X = \frac{a}{b + x}. \tag{6}$$

Further experiments with different thermocouple junction temperatures revealed that only $a$ changed and $b$ remained the same. He concluded that $a$ depended on the excitation force and $b$ on the length of the remaining parts of the circuit. As $X$ is proportional to the wire current $I_w$, (6) is of the form

$$I_w = \frac{E}{R_i + R_w} \tag{7}$$

in present-day notation, and represents Ohm's law in its original form. These experiments, and the empirical relationship

(6), appeared in 1826 in a paper entitled "Determination of the Law According to which Metals Conduct Contact Electricity, Together with the Outlines of a Theory of Volta's Apparatus and Schweigger's Multiplicator" [8]. Ohm's notebook of early 1826 shows [15] that in arriving at (6), he made the invalid assumption that the current moves on the surface of the conductor; this assumption was not mentioned in his paper, and Ohm must have recognized the invalidity of the assumption very shortly thereafter, because his subsequent theoretical work clearly states that the current is transported in the interior.

A subsequent paper [16] by Ohm contains the expressions

$$X = a\frac{kw}{l} \quad \text{and} \quad X = \frac{a}{L} \tag{8}$$

where $X$ is the (magnetic effect of) current, $w$ and $l$ are, respectively, the cross-sectional area and length of the wire, $k$ the "conducting power" of the metal, and $a$ depends on the "force." He called $L$ the "reduced length" of the circuit, a term which appears often in his work. (It is the hypothetical length of a wire of unit cross-sectional area and specific conductivity, having a resistance equal to the resistance of the circuit.)

## IV. THEORETICAL WORK ON OHM'S LAW

Contrary to popular belief, Ohm had published [17] some of his theoretical work shortly before writing his book. However, his book [9] encompasses all of this earlier work and is therefore examined here. The book contains two almost equal parts entitled "Introduction" and "The Voltaic Circuit," and a lengthy Appendix.

In the Introduction, Ohm introduces three basic postulates on which his theory is based. The first, on "the distribution of electricity within a body," assumes that the transfer of electricity takes place only from one particle directly to the next particle, in an amount proportional to the difference of potential between them. The other two postulates, which he calls "purely experimental," deal with "the mode of dispersion of electricity in the surrounding air" and "the appearance of electricity at the contact of two heterogeneous bodies." The first postulate is obviously the crucial one and comes from the theory of heat flow. Ohm points out the similarity with Fourier's and Poisson's work and acknowledges his debt to it.

Ohm begins by applying the postulates to a conducting ring of homogeneous material and constant cross section. His argument is essentially as follows. If a negligible amount of electricity escapes into the air (second postulate), each cross section must surrender as much electricity to one side as it receives from the other. From the first postulate, the amount depends on the difference of potential. Therefore, if the electricity flowing through the ring is the same everywhere, the potential must be linearly distributed along the length of the conductor, and Ohm plots it as a straight line. He then turns to the slope of the straight line. By considering similar rings of varying cross section or material, and requiring current continuity, he concludes that a section with reduced cross section or lower "ability to transfer electricity" (conductivity) must have a larger tension across it and, therefore, a larger potential gradient.

Having presented a geometrical method for determining the potential everywhere in the circuit, Ohm calculates the current from the piecewise linear potential distribution. He finds that "the magnitude of current in a galvanic circuit varies directly as the sum of all tensions and inversely as the entire reduced length of the circuit." Ohm's law is thus derived. After that he goes on to apply the results to special cases, including thermoelectric and "hydroelectric" (i.e., electrochemical) circuits. In particular, he shows that, beyond a certain point, increasing the number of turns in a galvanometer coil reduces galvanometer sensitivity owing to increasing "reduced length," and determines the rule for combining conductors in parallel.

The second part of the book is a more mathematical presentation of the essential results already given, including the solution of Laplace's equations for electrical conduction. Finally, the Appendix, "On the Chemical Power of the Galvanic Circuit," is devoted to circuits in which current is accompanied by chemical changes.

## V. CONTEMPORANEOUS ELECTRICAL SCIENCE

The state of electrical science prior to and during Ohm's work is of interest for several different reasons: 1) to determine whether Ohm's law had been anticipated by others, 2) to identify the earlier works which were necessary antecedents to Ohm's law, so that Ohm's contribution can be singled out, 3) to find the reasons for the poor reception of Ohm's work by his contemporaries and the delay of several years in his rewards, and, finally, 4) to understand the context, terminology, approach, and goals of Ohm's writings, experimental work, and theoretical investigations.

### *Prior Achievements*

The number of workers who studied the conduction of electricity in matter, or even in solids, prior to Ohm is rather large. These workers differed in the motivation of their work, the quantities observed, and the apparatus and methods used. Thus, Cavendish, Van Marum, Priestley, Children, and Harris used electrostatic discharge as the source of current, Cumming used a thermocouple as the source, while Barlow, Davy, and Becquerel used a Voltaic pile. Several of the "firsts" are summarized here.

The first study of conductivity is the unpublished work of Henry Cavendish (1731-1810) in England, around 1775, before steady currents were known. Cavendish attempted to relate the "resistance" (which should be taken to mean the potential difference in present-day terminology) to the "velocity" (to be translated as current) in a conductor by means of a power law; that the relationship should be a power law appears to be an *a priori* assumption. He discharged a Leyden jar through various lengths of a column of salt solution contained in tubes of different diameters. The intensity of discharge was judged by his sensation of electrical shock and was compared against that received from a standard tube. The index of the power law (determined to three significant figures!) was thus found to be approximately 1. His conclusion can be obtained from his experiment, only if the role of conductor length and diameter are known in advance, and Cavendish appears to have assumed (2) as being intuitively obvious without experimental proof. Furthermore, the experiments by Cavendish remained unknown to the world for a century until after Maxwell published them in 1879. This work obviously did not benefit Ohm; it is mentioned here because James Clerk Maxwell himself gave Cavendish the credit for having anticipated Ohm's law [18].

The first person to use steady current in studying conduction appears to be Peter Barlow (1776-1862) in England. Barlow used wires of different lengths and cross sections, placed across the same Voltaic pile, and determined the current by a galvanometer. He did not account for the internal resistance of his battery, and came to the conclusion that the current was approximately proportional to the square root of the length and to the inverse of the area of cross section.

The first person to establish experimentally the correct length and cross-sectional dependence in (2) was Sir Humphry Davy (1778-1829). Davy used an unusual method to avoid measuring any current. He connected a metallic conductor and an electrolysis cell in parallel across the same Voltaic pile, and adjusted the conductor length (i.e., the load on the pile) until the electrolysis of water in the cell could just begin, thus obtaining the same terminal voltage across the pile each time. He discovered that the wires of a given material, having the same ratio of length to area of cross section, were equivalent, i.e., produced the same loading.

Finally, the work of Antoine Becquerel (1788-1878) in France during the 1820's comes closest to Ohm's law expressed as (3), and was quoted by Ohm in his own writings. In Becquerel's experiment, two similar coils wound with wires of different diameter were connected in parallel but opposing each other's magnetic field. An additional length of wire was connected in series with the coil of thicker wire so as to increase its total resistance until the coils carried the same current as measured by a "multiplier" (essentially galvanometer). The additional length is thus a measure of the difference in the resistances of the two coil wires. Becquerel's work is significant for three reasons. First, he inadvertently devised a method that makes it unnecessary to account for the internal resistance of the battery, a major source of discrepancy in earlier works. Second, he experimentally verified the law of current continuity (that the total current is the same everywhere in a loop). Third, he showed that the conductivity of wires having the same ratio of length to cross section is the same and varies inversely with the length of the wire. Becquerel, however, did not arrive at (1).

### *Necessary Antecedents*

In his experimental work, Ohm did not use any recent developments or apparatus. He measured current by a torsion balance, rather than the relatively more modern galvanometer, as the galvanometers were less accurate then; indeed, a part of the paper [8] announcing the correct Ohm's law was devoted to an application of the law for improving the accuracy of galvanometers. The voltaic cell was also well established as a source. The thermoelectric source used in his later experiments was newer, but it may not have been absolutely necessary.

In his theoretical work, Ohm appears to have relied heavily on the theory of heat conduction, which had already been developed by Fourier and Laplace. Although that theory was essential for the theoretical derivation of Ohm's law and for suggesting concepts such as a continuous potential drop along a conductor, it cannot be called a necessary antecedent for Ohm's law. Ohm had already established (6) experimentally, before employing any theoretical considerations; his incorrect formulation of (5) is further evidence of the experimental origin of the law.

### *Terminology and Concepts*

Understanding Ohm's work is not straightforward because there is no clear one-to-one correspondence between the concepts or terms used then and now. A number of electrical phenomena had been discovered by the early nineteenth century, but the terminology was fluid and the concepts lacked quantitative definiteness. From electrostatics arose the terms "quantity of electricity" and "intensity of electricity," which may be translated as total charge and surface charge density, respectively. The most common instrument used to measure them was the gold-leaf (or pith-ball) electroscope, making mutual repulsion between charges the basic measure of charge. The current, on the other hand, was detected and measured in terms of its chemical or magnetic effects.

As the study of electric currents was still relatively new, it was cluttered with ideas from electrostatics, a subject that had long been studied. This is best illustrated by the distinction made between the terms "excitation force" (*erregende Kraft*) and "electric tension" (*elektrische Spannung*), both of which were used by Ohm. The "excitation force" was understood to cause the appearance of electric charges at the terminals of an open circuit battery. The "tension" of the battery, however, was taken to have vanished, as evidenced by electroscope leaves, when a conductor was connected across the battery. Studying current electricity with electrostatic concepts made it necessary to select the significant from the incidental parameters. For instance, Ohm himself repeated Davy's experiment, showing that the current in a circuit remained unchanged when a round conductor was replaced by a similar but flattened one, thereby establishing that it was the area of cross section and not the surface that influenced current flow.

The term "resistance" had been used loosely, and for more than one quantity, before being used for the constant of proportionality appearing in (1). Ohm himself used the term "reduced length" (*reducirte Länge*) everywhere in his book and adopted the existing term "resistance" (*Widerstand*) in his writings beginning in 1829, after recognizing the equivalence.

### *Reception of Ohm's Work*

There is difference of opinion among historians as to the reception of Ohm's work by his contemporaries. Some [19], [20] point out that many scientists, among them Fechner (1801-1887) and Lenz (1804-1865), began using Ohm's law in their work soon after it was published, and that Ohm's law was indeed well received with no unusual delays. Others believe that Ohm's work was not immediately appreciated, as evidenced by the criticism of this work, most notably by Pohl, which appeared in ***Berliner Jahrbücher für wissenschaftliche Kritik*** in 1828, and by the delay in both his university appointment and the translation of his book into English. These latter historians have proposed several reasons for the poor reception of Ohm's ideas.

Shedd and Hershey [13] suggest that the primary reason for ignorance of Ohm's work is the widespread impression that Ohm's law was entirely a theoretical deduction. This impression appears to have resulted from the fact that his theoretical book received a much wider circulation than his experimental papers, perhaps because there were numerous other uncoordinated, different and contradictory experimental results, expressed in ill defined terms and reported by other equally little known workers, from which Ohm's work could not be distinguished. Ohm, himself, did not sufficiently emphasize the experimental origin of his thoughts in his book, assuming either that the origin was well known (and, hence, the emphasis unnecessary) or that a purely theoretical deduction is a higher scholarly achievement (and is, therefore, more conducive to a university professorship). This explanation overlooks the fact that the two German journals in which Ohm published his experimental work had a wide circulation in the scientific community, which could hardly be matched by a book.

Winter [21] ascribes Ohm's failure in gaining immediate recognition primarily to the widespread faith in Hegelianism (*Naturphilosophie*) at the time in German universities. Hegelian thinking regarded mind as being capable of discovering the truth without the aid of matter, making laboratory experiments unnecessary. Ohm's work was therefore appreciated by few in (what is now) Germany where it was published. Indeed, the work was first appreciated outside Germany, and recognition came from outside–from Wheatstone, and the Royal Society of London. This explanation is weak because there are many counterexamples of the acceptance of other scientific works and of the reliance of German scientists on experimental methods regardless of Hegelianism.

Schagrin [22] is of the opinion that the rejection of Ohm's work resulted from the radical nature of his views which required a shift in the contemporaneous conceptual structure. The two parameters–voltage and current–which Ohm attempted to relate were very differently conceptualized. The concept of electric tension arose from electrostatics, whereas electric current was understood in terms of its magnetic action, and Ohm appeared to be confounding the distinction. This explanation has the same weakness as the last–there were several other accepted experimental results relating apparently very distinct parameters; magnetic effect of electric current and thermoelectric effect, themselves, are examples.

Thus, it seems that both the evidence of poor reception of Ohm's work, and the proposed explanations for it, are weak. The criticism received by Ohm appears to have a simple explanation: the obscurity of definitions and concepts then in use, and in Ohm's own writing. In his papers, Ohm's terminology is less than clear with regard to the "excitation force" or the "tension" (potential), and Ohm does not report having measured it. The delay in recognizing the significance of Ohm's law may simply have been the time necessary for the crystallization of the concepts involved.

## VI. OHM'S LAW IN RETROSPECT

The term "law" has been used in the sciences for a variety of types of results: from exact laws (like Coulomb's inverse square law in electrostatics) that are at present considered fundamental laws of nature, to approximate empirical relationships (like Boyle's law for gases) that apply under idealized conditions or over a limited range of parameter values. Ohm's law is also an empirical relationship, but it is applicable in a remarkably wide range of situations. If the qualifier "under isothermal condition" is added to the law, it is experimentally verifiable for metals from $\mathrm{pA/cm^2}$ to $\mathrm{gA/cm^2}$ [23], although some conflicting evidence is also available [24]. It is also possible to generalize Ohm's law to include the effect of temperature rise caused by current flow; the resulting current-voltage relationship then depends on the postulated mechanisms for heat loss and is nonlinear [25]. There are very few (for example, some biological) materials to which the low is not usefully applied.

Another characteristic of Ohm's law, uncommon among experimental laws, is the fact that under most circumstances, it has not required any refinements or correction factors for 150 years. To be sure, there are generalizations of Ohm's law, such such as those applicable to anisotropic media in which conductivity is a tensor rather than a scalar quantity. There are also some modifications of the law that extend the applicability of the law to a wider range of parameter values, materials, or operating conditions. For example, it may be applied to element semiconductors at high electric fields $E$ (but below the onset of avalanche breakdown, carrier injection, and intervalley transfer), provided the conductivity $\sigma$ is treated as a nonlinear function of current density $J$ [26]:

$$\frac{J}{E} = \sigma(J) = \sigma_o \left[1 - \left(\frac{J}{J_c}\right)^2\right] \tag{9}$$

where $\sigma_o$ is the low-field conductivity (in the limit $J \to 0$), and $J_c$ is the saturation value of the high-field current density (which is a function of crystal orientation, carrier density, and geometry). However, Ohm's law in its original form has a broad applicability.

In addition to the aforementioned direct generalizations of Ohm's law dealing with the flow of current under electrostatic fields, there are many "intellectual generalizations" applicable in other situations. For example, in semiconductors, where the current flow can occur both by drift and by diffusion of charge carriers, the diffusion current (given by Fick's law) can be included with drift current (given by Ohm's law) to obtain a modified Ohm's law [27], wherein the electrostatic potential is replaced by the electrochemical potential (also called the quasi-Fermi level). Similarities drawn from Ohm's law are also useful in many areas. An example is the linear relationship between magnetomotive force (MMF) and magnetic flux; the ratio of MMF to flux is called the "reluctance" of a magnetic path and is analogous to the resistance of an electrical path [28].

The limitations of Ohm's theoretical derivation of the law have become quite evident. The analogy between heat and charge, used by Ohm, is imperfect in some ways. In particular, when a thermal conductor transports heat, it also stores heat. By contrast, the electrical conductor transporting charge does not become charged. This distinction was never made by Ohm, but, fortunately for him, it does not alter the steady-state transport equations. In fact, Ohm's assumption that the resistance is independent of the electrical potential is more valid than Fourier's assumption that the thermal conductivity is independent of temperature.

Over the past century and a half, our understanding of the physical basis and the limitations of Ohm's law has improved considerably. This understanding also reveals the reasons for the success of Ohm's law. A microscope derivation of Ohm's law from Boltzmann's equation [29] shows that the linearity of the electric field-current density relationship is not easily violated in metals. A nonlinear correction term is necessary when, for example, the electric field changes significantly (in amplitude or direction) over the mean free path of carriers in metal [30]. A macroscopic derivation [31], on the other hand, shows that Ohm's law holds over *some* range of voltage $V$ and current $I$ for homogeneous isotropic materials, if we assume only the causality and analyticity of the $V$-$I$ relationship.

## REFERENCES

[1] H. Berring, "Do you really know Ohm's law," *IEEE Student J.*, vol. 5, pp. 33-35, Jan. 1967.

[2] M. Gorman, "Sir William Brooke O'Shaughnessy: Pioneer chemist in a colonial environment," *J. Chem. Educ.*, vol. 46, pp. 99-103, Feb. 1969.

[3] C. Susskind, "Henry Cavendish, Electrician," *J. Franklin Inst.*, vol. 249, pp. 181-187, Mar. 1950.

[4] E. G. Wever, *Theory of Hearing*. New York: Wiley, 1949.

[5] R. Appleyard, "Pioneers of electrical communication, VII–Georg Simon Ohm," *Elec. Commun.*, vol. 7, pp. 3-17, July 1928. Reprinted in R. Appleyard, *Pioneers of Electrical Communication*. London: Macmillan, 1930.

[6] G. S. Ohm, "Vorläufige Anzeige des Gesetzes, nach welchem Metalle die Contakt elektricität leiten; späterer Nachtrag," *J. Chemie Physik*, vol. 44, pp. 110-118, 1825; also in *Annalen der Physik*, vol. 4, pp. 79-88, 1825.

[7] —, "Uber Elektricitätsleiter," *J. Chemie Physik*, vol. 44, pp. 370-373, 1825.

[8] —, "Bestimmung des Gesetzes, nach welchem Metalle die Contakt elektricität leiten, nebst einem Entwurfe zu einer Theorie des Voltaischen Apparates und des Schweiggerschen Multiplicators," *J. Chemie Physik*, vol. 46, pp. 137-166, 1826.

[9] —, *Die galvanische Kette, mathematisch bearbeitet*. Berlin: Reimann, 1827.

[10] —, *The Galvanic Circuit Investigated Mathematically* (in German, W. Francis, transl.,) in *Scientific Memoirs, Selected from the Transactions of Foreign Academies of Science and Learned Societies, and from Foreign Journals*, R. Taylor, Ed., vol. 2, parts 7-8. London: Taylor, 1841, pp. 401-506; republished in the United States, T. D. Lockwood, Ed., New York: Van Nostrand, 1881.

[11] *Gesammelte Abhandlungen von G. S. Ohm*, E. Lommel, Ed. Leipzig, Germany: Meiner, 1892; also E. Lommel, "The scientific work of George Simon Ohm," (W. Hallock, transl.), in *Ann. Rep. Board Regents Smithsonian Institution, July 1891*. Washington, DC: Government Printing Office, 1893.

[12] L. Hartmann, *Aus Georg Simon Ohm handschriftlichem Nachlass. Briefe, Urkunden und Dokumente*. Munich, Germany: Bayerlandverlag, 1927.

[13] J. C. Shedd and M. D. Hershey, "The history of Ohm's law," *Pop. Sci. Monthly*, vol. 83, pp. 599-614, Dec. 1913.

[14] P. Hammond, "Georg Simon Ohm and his law," *J. Inst. Elec. Eng. (London)*, vol. 4, pp. 294-296, June 1958. Subsequent correspondence in vol. 4, pp. 435-436, Aug. 1958.

[15] J. L. McKnight, "Laboratory notebooks of G. S. Ohm–A case study in experimental method," *Amer. J. Phys.*, vol. 35, pp. 110-114, Feb. 1967.

[16] G. S. Ohm, "Versuch einer Theorie der durch galvanische Kräfte hervorgebrachten elektroscopischen Erscheinungen," *Ann. Physik*, vol. 6, pp. 459-469, 1826.

[17] —, Einige Elektrische Versuche," *J. Chemie Physik*, vol. 49, pp. 1-8, 1827.

[18] J. C. Maxwell, *The Electrical Researches of the Honourable Henry Cavendish, F.R.S., Written Between 1771-1781*. Cambridge, England: Univ. Press, 1879, p. 333.

[19] N. Reingold, Ed., *The Papers of Joseph Henry*, vol. 2. Washington, DC: Smithsonian Institution, 1972, pp. 299-300.

[20] K. L. Caneva, "Ohm, Georg Simon," in *Dictionary of Scientific Biography*, vol. 10, C. C. Gillispie, Ed. New York: Scribner's, 1974.

[21] H. J. J. Winter, "The reception of Ohm's electrical researches by his contemporaries," *Phil. Mag.*, vol. 35, pp. 371-386, June 1944.

[22] M. L. Schagrin, "Resistance to Ohm's law," *Amer. J. Phys.*, vol. 31, pp. 536-547, July 1963.

[23] H. Heinrich and W. Jantsch, "Verification of Ohm's law up to current densities of $10^9$ A/cm$^2$ by sub-nanosecond pulse measurements," *Solid State Commun.*, vol. 7, pp. 377-379, Feb. 1968.

[24] H. Jager, U. Seydel, and H. Wadle, "Deviations from Ohm's law for a metal at high-current densities," *Phys. Lett. A*, vol. 55, pp. 481-482, Feb. 9, 1976.

[25] L. Gador, "Thermal nonlinearity," *Acta Tech.*, vol. 81, no. 3-4, pp. 299-312, 1975.

[26] R. Jaggi, "Deviations from Ohm's law in semiconductors," *J. Phys. Chem. Solids*, vol. 29, pp. 1699-1702, Sept. 1968.

[27] B. G. Streetman, *Solid State Electronic Devices*. Englewood Cliffs, NJ: Prentice-Hall, 1972, p. 125.

[28] P. E. K. Donaldson, "Applying magnetic Ohm's law to permanent magnets," *Wireless World*, vol. 81, p. 567, Dec. 1975.

[29] J. M. Ziman, *Principles of the Theory of Solids*. Cambridge, England: Cambridge Univ. Press, 1971, p. 181.

[30] R. H. Havemann, P. F. Engel, and J. R. Baird, "Nonlinear correction to Ohm's law derived from Boltzmann's equation," *Appl. Phys. Lett.*, vol. 24, pp. 362-364, Apr. 15, 1974.

[31] B. Podolsky and H. H. Denman, "A macroscopic approach to Ohm's law," *Amer. J. Phys.*, vol. 34, pp. 814-816, Sept. 1966.

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