Wide-Angle Dispersion of High-Frequency Sound

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A description of the design of a series of high-frequency horns with improved performance in the horizontal plane and a minimum of diffraction in the vertical plane.
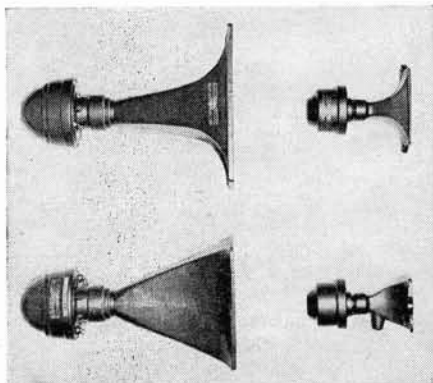

Fig. 1. Two commercial horns which employ the reverse-flare principle described.

Summary:

A high degree of wide-angle dispersion in one plane may be achieved by means of a compound horn in which the primary section allows wave expansion only into the undesired plane. This creates a pressure gradient along the horn walls defining the desired plane. The built up pressure is subsequently allowed to expand into the desired plane by reversal of the direction of flares of the horn walls. This reverse-flare device in conjunction with a square horn mouth provides wide-angle radiation free from phase cancellation and mouth diffraction effects.

AN ESSENTIAL REQUISITE of a tweeter radiator is that the high-frequency energy be distributed over a wide horizontal angle. Failure to attain such a characteristic results in off-axis loss of level of the high frequencies. How-

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ever, wide-angle dispersion by itself is not the optimum solution to high-frequency radiation. The following three attributes must prevail simultaneously.

a. Maintenance of the sound pressure of the entire frequency range of the tweeter over the entire angular coverage desired.
b. The over-all efficiency of the horn must not be sacrificed in making the horn a wide-angle radiator.
c. The angular response shall be free

of irregular energy lobes as the frequency changes.

Figure 2A compares graphically these characteristics of merit for the three types of tweeter horns commonly known as the multicellular type, the pie-wedge type, and the recent University "Reverse Flare" type. The multicellular characteristic shows regions of extreme energy fluctuation. Such fluctuation, dependent upon frequency and angle, is
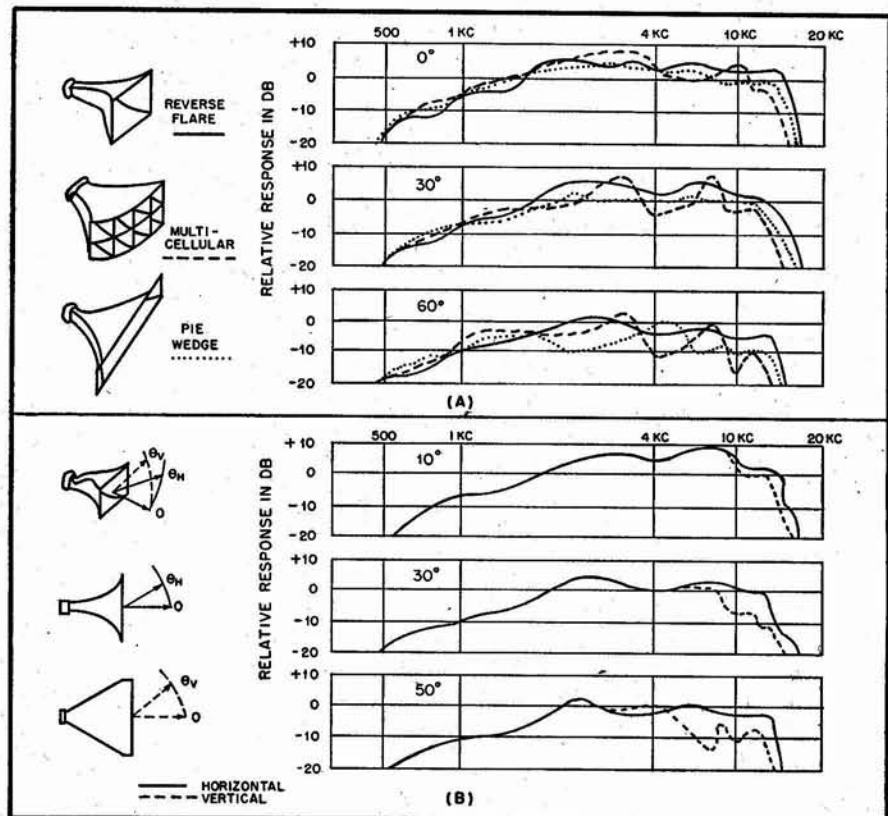

Fig. 2. (A) Comparison of performance of reverse flare, multicellular, and pie-wedge horns. (B) Relative angular response in horizontal (solid lines) vs. vertical (dotted lines) radiation in reverse-flare horn.

due to the phase cancellation of energy from the several individual sources of radiation from the multicellular mouths. Where multicellular sources are absent, as in the pie-wedge type of horn and in the reverse flare horn, there is no evidence of these phase discriminatory cancellations. However, it will be noted that the pie wedge exhibits considerable energy loss in the mid-band-pass region for increasing angle. The conventional narrow vertical dimension of the mouth of this type of horn gives rise to excess diffraction of energy into the vertical plane which is tantamount to energy loss in the horizontal plane.

Where neither multicellular nor wedge-mouth devices are used, as in the reverse flare horn, high horizontal efficiency performance is obtained, and there are no phase cancellation regions, as *Fig.* 2A indicates. The favorable ratio of horizontal to vertical radiation for this type of horn is illustrated in *Fig.* 2B.

The reverse flare horns illustrated in *Fig.* 1 show that this type of horn first expands rapidly in the vertical direction, and as the mouth of the horn is approached it begins to expand also in the horizontal direction. Hence the name "Reverse Flare" horn. The second feature of this horn is its square mouth configuration (in contrast to the narrow slit type). This *combination* of reverse flare and square mouth shape prove especially effective in giving the horn its desirable performance.

It is often stated that high frequencies project in a narrow beam, while low frequencies spread out in a wide beam. The truth of the matter is that all sound spreads out in ever widening circles regardless of frequency, *provided* there is no restriction or obstruction in the way. For instance, a pulsating sphere will give rise to a perfectly spherical wave front regardless of the frequency of pulsation. On the other hand, a twelve-inch cone type speaker beams the higher frequencies because of the radiation characteristics of the diaphragm shape and construction. It is the combination of *radiator size, radiator configuration, and frequency* which determines how loosely or how tightly a particular high-frequency sound will be dispersed. Following the same laws of physical acoustics, the *horn size and shape* will play an im-

portant part in the dispersion of the high-frequency energy radiated from the mouth of the horn.

Horn Pressures

All horns, including the reverse flare type, are pressure controlling devices. At the throat of the horn (the narrow end), the sound pressure is the greatest, while at the mouth of the horn (the wide open end), the sound pressure is the least. At any point along the axis of the horn between the mouth and the throat, the sound pressure is of some intermediate value. The actual manner in which the sound pressure throughout the horn varies depends upon the rate of growth of the area of cross-section of the horn. This cross-sectional growth is determined by the law under which the particular horn expands.

One of the most efficient of horn expansions is the exponential type. This horn is remarkably efficient as an impedance matching device between the source of sound at the throat of the horn and the atmosphere into which the mouth of the horn radiates. Its cross-sectional area expands according to the "natural law of growth," expressed by

$$\frac{A_2}{A_1} = \epsilon^{.000366 f_{co} x}$$

where f_{co} = the designated cut-off frequency of the horn in cycles per second

x = the distance in centimeters between two points within the horn and along the horn axis

A_2, A_1 = the areas of cross-section of the horn at the two designated points along the axis

ϵ = base of natural logarithms, 2.71828.

By choosing the cut-off frequency f_{co} desired, the physical expansion of the horn may be laid out. From this equation it is seen that for a given distance x between two points, and for low values of cut-off f_{co}, the exponential factor ϵf_{co} is smaller than for large values of cut-off frequencies. Consequently, the ratios of areas at these points along the axis are smaller for low-cut-off horns than for high-cut-off horns. This means

that the horn designed for low cut-off expands slowly, while the horn designed for high cut-off expands rapidly.

This provides a means of determining in what geometric manner the sound pressures within the horn are distributed. For instance, in a horn with a high cut-off frequency which will necessarily expand and flare out quickly, the fast flaring walls of the horn allow the wave front to spread out rapidly in a direction transverse to the axis. Consequently, the total pressure of the wave front will be distributed quickly over the enlarged area between the fast flaring walls. This will result in a rapidly diminishing pressure per square unit of the wave front surface. On the other hand, in a horn with a low cut-off frequency where the walls expand slowly, the wave front development in a direction transverse to the axis will be restricted. Accordingly, the per unit area pressure distribution on the wave-front surface will diminish slowly. Thus a knowledge of the cut-off frequency of the horn will determine the
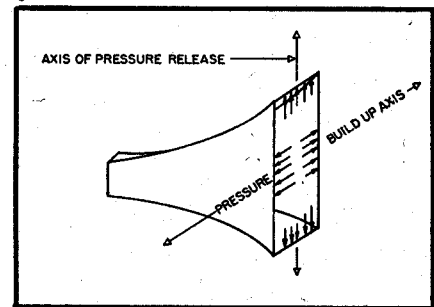

Fig. 4. Internal horn pressure configuration. The flaring upper and lower walls allow wave-front expansion which reduces the pressure on these walls. The non-flaring vertical walls restrict wave expansion causing build up of pressure against these walls.

fashion in which the sound pressures in the horn are geometrically controlled.

Sound Radiation

To perform useful acoustic work, these internal sound pressures must eventually emerge from the horn into space. In making this transition, the sound pressures cross the mouth of the horn, which is its threshold into space. The mouth of the horn then becomes in essence the *sound radiator* for the energy reaching it with a particular pressure variation dependent upon the horn flare. Thus the size and configuration of the horn mouth, the horn flare, and the frequency involved will determine the degree of wave-front dispersion. Standard functional analysis of exponential horns shows that in this combination of factors, fast flare means wide-angle dispersion for highs, and large mouth means narrow-angle dispersion for lows (provided the mouth diameter is at least one third the wave length of the sound being radiated).

In the practical application of these guiding factors of horn design, the reverse-flare horn finds it possible to strike a unique balance between horn flare and mouth size. In brief, this is accomplished as follows. By first restricting wave expansion in one plane
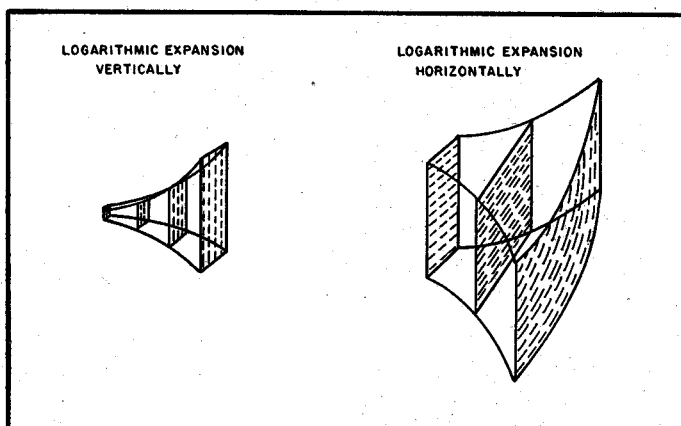

Fig. 3. Cross-sectional area development of reverse - flare horn. One area progresses from the preceding area according to the exponential expansion for a given cut-off frequency.

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and then suddenly releasing the wave front in that very same plane, it becomes possible to flare the horn more suddenly near its mouth, which will result in wider dispersion. Second, by choosing the point at which wave front restriction changes *direction*, it is possible to arrive at a vertical mouth opening of sufficient size to control the vertical directivity as desired.

Since the angular dispersion is a partial function of the geometric distribution of pressure within the horn, the reverse-flare horn must be examined geometrically. *Figure* 3 indicates the cross-sectional area distribution of the reverse-flare horn. The horn is, at all times, exponential. Regardless of the shape of the cross-section, one area progresses from the preceding area in a logarithmic fashion with distance x, bounded strictly by the cut-off frequency for which the horn is designed. Since these areas are the product of the vertical and the horizontal dimensions, the horizontal dimension may be chosen to have any desired value. This will permit primary restriction of the wave expansion to be made as severe as desired. If the horizontal dimension is to be kept unchanged from the smallest value as found at the throat of the horn, it will simply mean that the vertical height will do all the expanding. Or if it is desired to let the horizontal dimension expand slightly, then the vertical height need not expand as rapidly. The cross-sectional area may be proportioned in any way best suited to the desired result.

The purpose for the present is actually to *restrict* early horizontal spread of the wave front *within* the horn, but to allow the wave front to expand freely in the vertical direction, within the horn. Restriction of the wave front expansion from the horizontal plane gives rise to an area of high wave-front pressures against the restraining vertical walls. Conversely, freedom of the wave to expand in the vertical direction will result in minimum wave-front pressures against the fast flaring upper and lower walls. As an analogy, picture a cylindrical tube in which two inserted pistons confine a gas under pressure. When these pistons are stationary, the confined gas pressure is equally distributed over the walls of the tube and the piston faces. If the pistons are now suddenly pulled apart, the confined gas pressure will tend to expand toward the piston faces. If the pistons retreat fast enough, the expanding gas may never catch up to them. Consequently, these retreating pistons, which allow the gas to expand in their direction, will have minimum pressure exerted upon them, whereas the restraining walls of the tube will experience the greater gas pressure.

Returning to the horn walls, the difference in wave front pressures against the vertical walls and the top and bottom walls may be represented graphically, as shown in *Fig.* 4. This pressure configuration represents the state of affairs concerning the i n t e r n a l pressures against the horn walls extending from the throat of the horn down to the point at which it may be desired arbitrarily to reserve the pressure distribution. For the moment, let any such point be chosen along the horn axis at which to introduce this pressure reversal.

Fig. 5. Flare reversal of the horn walls turns the direction of the wave-front expansion from vertical to horizontal.

Such a desired pressure reversal may be accomplished by restricting the flare of the upper and lower walls which have been expanding, simultaneously flaring out the vertical walls which up to now have not been flared. This will permit the side thrust pressures which have been built up against these vertical walls, to suddenly burst out of their restraining shell, so to speak, and to be impelled actively along the direction of the new flare. This action is illustrated in *Fig.* 5. In Section *A* of the horn all the wave expansion is in the vertical direction, with wave restriction in the horizontal direction. At these restraining walls there is a resultant boundary of increased pressure. After section X-X, Y-Y, the portion of the horn indicated as *B* now flares rapidly in the horizontal direction, while in the vertical direction there is practically no flare. Because the wave front can no longer expand vertically, pressure will be built up against these upper and lower walls. On the other hand, since the wave is allowed to expand horizontally, the pressure will dissipate itself over the horizontal angle. Thus, the pressure reversal has been accomplished.

Because of this pressure reversal, the dispersion gain in the horizontal direction is two-fold. The first element of gain obtains from the manner that the *B* section of the horn is energized. It is quite permissible to consider section Y-Y as being the throat for a new horn *B*, and that the sound source which feeds this horn to be the mouth X-X of horn *A*. This new sound source X-X is one in which the driving pressure is greatest in the direction of ultimate dispersion, that is, in the horizontal direction. Thus the sound source is "matched" on a pressure configuration basis to the shape of the horn which is to disperse that pressure. Increased pressure dispersion is thus obtained as compared with the case where the throat of the horn is fed from a source of symmetrical pressure distribution.

The second element of gain stems from the much greater flare possible near the mouth of the horn, where it counts most. By allowing but little horizontal expansion prior to the point of pressure reversal, and suddenly limiting the vertical expansion severely after the point of expansion, it becomes possible
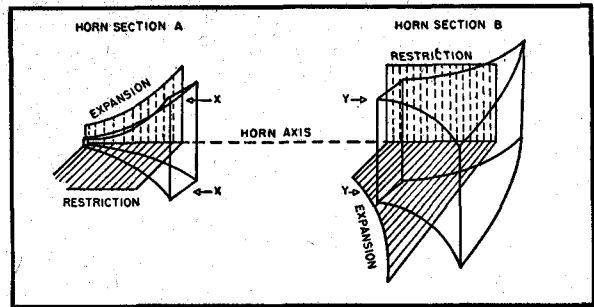
thereafter to throw all the desired exponential expansion into the horizontal direction near the mouth end of the horn. As a result of this terminal expansion, the horn will expand much faster in the horizontal direction than would be the case if the horizontal expansion were to have started back at the original horn throat. The end effect of this faster physical flare for the new short horn section is an increased dispersion angle for the high frequencies.

Having thus accomplished wide-angle dispersion of the radiated sound into the horizontal direction by means of flare control, one additional step may be taken to prevent undue dispersion or diffraction effects into the vertical direction. Such control may be obtained in two ways. Just as low flare will produce relatively narrow dispersion, so a large mouth will reduce diffraction. We already have a slow flare in the vertical direction because of the vertical flare restriction near the mouth of the horn. Consequently, half the problem of restricting the vertical dispersion is already taken care of. The second controlling factor, that of mouth size in the vertical direction, may be obtained if that dimension is permitted to become at least one-third of the wavelength of the lowest frequency to be radiated.

Such a condition is accomplished in these reverse-flare horns by locating the point of flare reversal where the vertical dimension is close to the desired vertical mouth height, and from this point on providing but slight vertical expansion. By such dimensional manipulation it is possible to terminate the horn in a square horn mouth, the vertical height of which is sufficiently large to reduce diffraction in the vertical plane to a minimum.

The end result of this flare reversal may be summed up as follows: The mouth end of the horn is transformed into a much faster flare in the horizontal direction; the pressure configuration within the horn is altered to match the flare distribution; the vertical mouth size is proportioned to provide minimum diffraction and dispersion. This three-way attack on the problem of high-frequency distribution produces a family of horns which exhibit a high degree of dispersion efficiency.